On the Time Inconsistency of International Borrowing in an Optimal Growth Model

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Abstract

This paper analyzes international borrowing and lending in an optimal growth model with adjustment costs for investment. We study the relation between optimal savings and investment, and the current account, in the transition towards the balanced growth path, and derive the implications of financial openness for both the transition path and the balanced growth path. A comparison with financial autarky reveals that, to the extent that countries start from different initial conditions, and there is pre-commitment to debt repayment, financial openness is beneficial for both “poor” and “rich” countries, as it allows them to engage in mutually beneficial inter-temporal trade. During the adjustment process, relatively “poor” countries experience higher consumption and investment compared to autarky, and thus cumulate current account deficits. There is an inter-temporal tradeoff, in that they experience lower steady state consumption, due to the need to service their accumulated foreign debt. The opposite happens in relatively “rich” countries. The inter-temporal tradeoffs implied by financial openness result in a time inconsistency problem. “Poor” countries reach a point in the adjustment process at which it is welfare improving to renege on their commitment to repay their foreign debt. In the absence of sufficient pre-commitment mechanisms, international lenders anticipate these incentives, and international borrowing and lending are driven to zero. This time inconsistency problem can thus explain both the Feldstein-Horioka puzzle and the Lucas paradox that capital does not flow from “rich” to “poor” countries. Credible sanctions in the case of default and ceilings on international borrowing are analyzed as partial solutions to the time inconsistency problem of international borrowing.

Keywords: time inconsistency, international borrowing, optimal growth, financial openness, debt default

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Email: alogoskoufis@me.com Web Page: https://alogoskoufisg.com .
The neoclassical optimal growth model of Ramsey (1928), Cass (1965) and Koopmans (1965) is the workhorse of both growth theory and, in its stochastic version, real business cycle theory. Yet its use in international economics has been relatively limited, due to the assumption of the canonical optimal growth model that investment is determined by domestic savings. In order to determine the path of the current account in an open economy growth model one needs an investment function which is independent of the savings function. It is only in this case that one can analyze the convergence process and the evolution of the difference between savings and investment, and thus the current account, in growing open economies.\footnote{For example, Solow (2005), reflecting on 50 years of growth theory, says, “It is also a little odd (italics added) that there was not more in the way of open-economy growth modeling” p. 4. A remark of Barro and Sala-i-Martin (1995), ten years earlier, gives a clue about a potential explanation of the Solow oddity: “We began with the seemingly straightforward task of extending the Ramsey model to an open economy by allowing for international borrowing and lending. This extension led however to some counterfactual results: convergence speeds for the capital stock and output were infinite and, except for the most patient country, consumption (per unit of effective labor) tended to zero and assets became negative.” (p.127).}

The neoclassical optimal growth model has been used in a number of studies of open economies, but few of these have introduced an independent investment function. The early paper of Bardham (1967) relies on the canonical optimal growth model. The closed economy neoclassical optimal growth model, augmented by the $q$ theory of investment, which results in an independent investment function, was first analyzed by Abel and Blanchard (1983). A small open economy version was first analyzed by Blanchard (1983) and Blanchard and Fischer (1989), Chapter 1. While in the canonical neoclassical optimal growth model convergence and the optimal level of foreign borrowing are achieved instantly, and there is no convergence process or further current account dynamics, in the augmented model there is both gradual convergence and current account dynamics.\footnote{See Barro and Sala-i-Martin (1995) Ch.3 for open economy versions of the canonical optimal growth model, and the infinite speed of convergence result. Lucas (1990) uses the standard neoclassical growth model to highlight the famous Lucas paradox, of why capital does not flow from rich to poor countries and discusses potential solutions. Barro, Mankiw and Sala-i-Martin (1995) examine limited capital mobility in a neoclassical growth model with human and non-human capital, but without adjustment costs for investment. Cohen and Sachs (1986) analyze an endogenous growth model with the possibility of debt repudiation. Obstfeld and Rogoff (1996) and Vegh (2013) survey and present alternative open economy models based on the inter-temporal approach to the current account that have been developed since the 1980s, using the standard neoclassical growth model, mostly without an independent investment function.}

This paper re-examines the augmented neoclassical optimal growth model, to study the convergence process and the balanced growth path of financially open economies, both in the presence of pre-commitment and in the absence of pre-commitment by international borrowers. The analysis relies on a neoclassical optimal growth model with adjustment costs for investment of the Abel and Blanchard (1983) form. Households choose individually optimal consumption plans, and firms choose individually optimal investment (and employment) plans, as postulated by the $q$ theory of investment. It is shown that under financial openness the optimal policy is time inconsistent for relatively “poor” economies, and that, in the absence of commitment mechanisms, this time inconsistency can explain anomalies such as the Feldstein Horioka (1980) puzzle and the Lucas (1990) paradox of why capital does not flow from “rich” to “poor” countries, and “poor” countries get trapped in a regime financial autarky.

Under financial autarky domestic savings are continuously equal to domestic investment. Under financial openness savings and investment generally differ, and their difference affects the speed of convergence and determines the dynamic path of the current account.
We first analyze the relation between growth and the current account in the transition towards the balanced growth path, and derive the implications of financial regimes autarky and financial openness for both the convergence process and the balanced growth path.

We analyze both the case of a small open economy, and the case of a two country world. We distinguish between two types of economies, a “poor” and a “rich” economy. A “poor” economy is an economy with a “low” initial capital stock, relative to its steady state capital stock, and a “rich” economy is an economy with a relatively “high” initial capital stock. Thus, for a “poor” economy, the initial equilibrium real interest rate under autarky is higher than the initial world real interest rate it faces under financial openness. The opposite applies for a “rich” economy.

For both types of economy, there are potential gains from financial openness, as long as their initial “autarky” real interest rate differs from the world real interest rate, and there is pre-commitment on the part of the international borrower not to default. In such a case, financial openness implies enhanced opportunities for inter-temporal trade. A “poor” economy can increase welfare by trading higher consumption during the convergence towards the steady state, for lower consumption in the steady state itself. A “rich” economy can increase welfare by trading lower consumption during the convergence process, for higher consumption in the steady state. This inter-temporal trade takes place through the trade balance and the current account.

Under pre-commitment, a “poor” economy achieves higher consumption and investment during the adjustment path to the steady state under financial openness than under autarky. As a result, it experiences transitional but persistent current account deficits, accumulating foreign debt. In the steady state, which is approached at a higher speed of convergence, because of the higher interim investment rates, it returns to external balance. However, the external debt it has accumulated results in lower steady state consumption compared to financial autarky. The benefits of financial openness arise because of higher consumption during the transition path relative to autarky, but these benefits are partly reversed due to lower consumption on the balanced growth path. Inter-temporal trade thus also implies inter-temporal tradeoffs.

Under pre-commitment by the “poor” economy, a “rich” economy will experience lower consumption and lower investment relative to financial autarky during the adjustment path. It will thus experience transitional but persistent current account surpluses, accumulating positive net foreign assets. In the steady state it returns to external balance, but the interest payments on the external assets it has accumulated will result in higher consumption compared to financial autarky. The benefits of financial openness arise because of higher consumption during the balanced growth path relative to autarky. However, these benefits have been partly pre-paid, through lower consumption on the adjustment path. Inter-temporal trade thus implies the opposite inter-temporal tradeoffs relative to the “poor” economy, although, overall, there are welfare gains for a “rich” economy too.

Thus, both the adjustment paths and the balanced growth paths differ across the two alternative regimes of financial autarky and financial openness. On the balanced growth path, capital and...
domestic output (GDP) per capita are the same under the two alternative financial regimes of autarky and openness. So is the steady state real wage and the real interest rate. However, there are significant differences under the two alternative regimes for steady state national income (GNP) and consumption per capita. These differences arise because of the dynamics of the current account and the accumulation of net foreign assets during the adjustment path to the steady state. To the extent that a country has accumulated foreign debt, steady state domestic output (GDP) is higher than steady state national income (GNP), as the country has to pay interest on its foreign debt. To the extent that a country has accumulated positive net foreign assets, steady state domestic output (GDP) is lower than steady state national income (GNP), as the country receives interest on its foreign debt. Thus, the initial capital stock has implications for steady state consumption and the relationship between gross domestic product (GDP) and gross national income (GNP) per capita on the balanced growth path. Steady state domestic output (GDP) will be higher than steady state national income (GNP) for an initially “poor” economy, and the opposite will happen for an initially “rich” economy.

The analysis of financial openness thus suggests that, under pre-commitment, capital should flow from “rich” to “poor” countries, and that the Feldstein Horioka (1980) puzzle and the Lucas (1990) paradox should not exist. We next move on to explain these anomalies.

The explanation is in terms of the time inconsistency of the optimal policy for a “poor” economy. We prove that the ex ante optimal path under financial openness is time inconsistent for a “poor” economy. This is the case because the benefits of financial openness are front loaded for a “poor” economy and the costs are back loaded. The opposite is the case for a “rich” economy. Thus, there comes a point along the adjustment path at which, for the “poor” economy, the net benefits from reneging on the ex ante optimal policy, and resorting to debt repudiation and nationalization of foreign capital, are greater than the remaining net benefits of sticking to the ex ante optimal policy. In the absence of pre-commitment mechanisms, the point where debt repudiation becomes the ex post optimal policy is only a matter of time and arrives before the “poor” economy reaches the balanced growth path.

Thus, to the extent that sufficient pre-commitment mechanisms do not exist, international lenders in the “rich” country anticipate the risk of repudiation, and are only prepared to lend under conditions that compensate them for that risk. The time consistent equilibrium is thus financial autarky, or, if credible sanctions can be invoked, international lending and borrowing restrictions.

Capital market restrictions for international borrowers thus become endogenous to our analysis. They can be viewed as the optimal response of international lenders to the incentives of international borrowers to repudiate on their foreign debt, since the optimal policy of international borrowers is not time consistent.

We demonstrate that if the sanctions that can be credibly imposed in the case of default are limited, financial openness will necessarily imply borrowing constraints for a “poor” country. The period of accumulating foreign debt will be shorter, there will be an upper limit on external debt, and the poor country will never have an incentive to default, unless there is an unanticipated change in the fundamentals.

The pros and cons of financial openness are hotly debated. Obstfeld (1998) and Henry (2007) survey the literature on the pros and cons of financial openness, while Gourinchas and Jeanne
(2006) use a standard neoclassical growth model, albeit without investment adjustment costs, to try and estimate the welfare benefits of financial openness for growing economies.

Obstfeld concludes that “Despite periodic crises, global financial integration holds significant benefits and probably is, in any case, impossible to stop—short of a second great depression or third world war. The challenge for national and international policymakers is to maintain an economic and political milieu in which the trend of increasing economic integration can continue.” (p. 28). Henry concludes that “There is little evidence that economic growth and capital account openness are positively correlated across countries. But there is lots of evidence that opening the capital account leads countries to temporarily invest more and grow faster than they did when their capital accounts were closed.” (pp. 928-929).

On the other hand, Gourinchas and Jeanne (2006) conclude that “developing countries do not benefit greatly from international financial integration in a calibrated neoclassical model.” (p.736).

Thus, there are a number of conflicting conclusions and views, for the assessment of which the present paper provides a consistent framework.

The rest of the paper is organized as follows: In section 1 we present the basic representative household model and characterize its optimal consumption plan. In section 2 we analyze the optimal production, employment and investment decisions of firms, under the assumption of competitive markets and convex (quadratic) adjustment costs for investment. Equilibrium under financial autarky is analyzed in section 3. In section 4 we analyze a small open economy under financial openness and pre-commitment, and present our main conclusions for a small open economy. In section 5 we present the model of a two country world, with otherwise similar economies that differ only in their initial capital stocks. We present numerical simulations of these models in Appendix A and calculate the welfare benefits of financial openness which appear to be quantitatively small. In section 6 we discuss the time inconsistency problem under lack of pre-commitment mechanisms, and alternative solutions such as potential sanctions and ceilings to international borrowing. The last section summarizes our conclusions.

1. Optimal Consumption in the Representative Household Model

The economy is assumed to consist of an infinite number of households, indexed by \( j \), where \( j \in [0,1] \). Household \( j \) chooses a consumption path to maximize,

\[
U_j = \int_{t=0}^{\infty} e^{-(\rho-n)t} \ln c_j(t) dt
\]  

(1)

subject to the instantaneous budget constraint,

\[
a_j(t) = (r(t)-n)a_j(t) + w_j(t) - c_j(t)
\]  

(2)

and the household’s solvency (no-Ponzi game) condition,

\[
\lim_{t \to \infty} \int_{s=0}^{t} e^{-(\rho-n)s} ds \cdot a_j(t) = 0
\]  

(3)
$\rho>0$ is the pure rate of time preference, $n>0$ is the exogenous rate of growth of household members (and population), $c_j(t)$ is the per capita consumption of household $j$ at instant $t$, $a_i(t)$ denotes per capita non-human wealth of household $j$ at instant $t$, and $w_j(t)$ is per capita non asset (labor) income of household $j$ at instant $t$. $r(t)$ is the real interest rate. Instantaneous utility is assumed logarithmic, implying that the elasticity of inter-temporal substitution is equal to unity. We also assume that $\rho-n>0$, which is necessary for (1) to be finite.\footnote{The results can be generalized to the case where the elasticity of inter-temporal substitution differs from unity. Since we do not focus on the role of this parameter, we maintain the simpler specification of logarithmic preferences throughout.}

Integrating (2), using the solvency condition (3), and assuming that the initial per capita non-human wealth of the household is equal to $a_j(0)$, yields the familiar inter-temporal budget constraint, that the present value of per capita consumption must equal the present value of per capita labor income plus initial per capita non-human wealth.

$$a_j(0) + \int_{t=0}^{\infty} w_j(t)e^{-\int_{s=0}^{t} (r(s)-n)ds} \, dt = \int_{t=0}^{\infty} c_j(t)e^{-\int_{s=0}^{t} (r(s)-n)ds} \, dt$$

Maximization of (1) subject to (2) and (3) yields the familiar Euler equation for consumption,

$$c_j(s) = (r(s) - \rho)c_j(s)$$

We can aggregate the first order condition (5) to derive aggregate consumption, as,

$$\dot{C}(t) = (r(t) - \rho + n)C(t)$$

where $C$ is aggregate consumption of goods and services.

2. Production, Employment and the Investment Decisions of Firms

Producers are competitive firms, employing capital and labor to produce a homogeneous commodity. Firms are indexed by $i$, where $i \in [0, I]$. The production function of firm $i$ at time $t$ is assumed Cobb Douglas with constant returns to scale, and is given by,

$$Y_i(t) = AK_i(t)^\alpha \left( h(t)L_i(t) \right)^{1-\alpha}$$

where $Y$ is output, $K$ physical capital, $L$ the number of employees and $h$ the efficiency of labor. The efficiency of labor is the same for all firms. $A>0$, which measures total factor productivity, and $0<\alpha<1$ are exogenous technological parameters.

We assume that the efficiency of labor grows at an exogenous rate $g$, which measures the rate of technical process. We thus assume that,

$$h(t) = e^{gt}$$
where $g$ is the rate of exogenous (labor augmenting) technical progress and the efficiency of labor at time 0 has been normalised to unity.

Substituting (8) in (7) and aggregating across firms, we have,

$$Y(t) = AK(t)^\alpha \left(e^{\alpha t} L(t)\right)^{1-\alpha} \quad (9)$$

In order to determine the production, employment and investment decisions of firms we first define the instantaneous profit function of firm $i$. This is given by,

$$Y_i(t) - w(t)L_i(t) - \left[1 + \frac{\phi}{2} \left(\frac{I_i(t)}{K_i(t)}\right)\right]I_i(t) \quad (10)$$

where $w$ is the real wage and $\phi$ is a positive constant measuring the intensity of the marginal adjustment cost of gross investment $I$. The relation between gross and net investment is given by,

$$I_i(t) = K_i(t) + \delta K_i(t) \quad (11)$$

Each firm thus chooses an employment and an investment plan to maximize,

$$\int_{s=t}^\infty e^{-r(s)} \left[Y_i(s) - w(s)L_i(s) - \left[1 + \frac{\phi}{2} \left(\frac{I_i(s)}{K_i(s)}\right)\right]I_i(s)\right] ds \quad (12)$$

subject to the production function (7) and the accumulation equation,

$$\dot{K}_i(s) = I_i(s) - \delta K_i(s) \quad (13)$$

Since firms are competitive, they take the path of real wages and real interest rates as exogenously given.

From the first order conditions for the maximization of (12) subject to the production function (7) and the capital accumulation equation (13), we get,

$$w(t) = (1-\alpha)A \left(\frac{K_i(t)}{L_i(t)}\right)^\alpha h(t)^{1-\alpha} \quad (14)$$

$$q_i(t) = 1 + \phi \left(\frac{I_i(t)}{K_i(t)}\right) = 1 + \phi \left(\frac{\dot{K}_i(t)}{K_i(t)} + \delta\right) \quad (15)$$

$$\left[r(t) + \delta - \frac{q_i(t)}{q_i(t)}\right]q_i(t) = \alpha A \left(\frac{K_i(t)}{L_i(t)}\right)^{\alpha-1} h(t)^{1-\alpha} + \phi \left(\frac{\dot{K}_i(t)}{K_i(t)} + \delta\right)^2 \quad (16)$$

where $q$ is the shadow price of installed physical capital.
These first order conditions have well known interpretations. (14) states that firms will hire labor until the marginal product of labor is equal to the real wage. (15) is the condition linking the shadow price of installed capital \( q \) to the gross investment rate. (16) states that the user cost of capital (on the left hand side) is equal to the marginal product capital, which consists of the marginal product of capital in current production, plus the reduction of future adjustment costs for investment.

The path of investment and capital must also satisfy the transversality condition,

\[
\lim_{s \to \infty} e^{rs} q_j(s) K_j(s) = 0
\]  

(17)

Note that firms take the efficiency of labor as exogenously given. Also note that because the real wage is the same for all firms, and all firms share the same technology, all firms will choose the same capital labor ratio. Since the real interest rate is the same for all firms, and all firms share the same technology, all firms will also choose the same gross investment rate.

Aggregating (14)-(16) across firms, and using (8), the aggregate first order conditions are given by,

\[
w(t) = (1 - \alpha) A k(t)^\alpha e^{\gamma t}
\]  

(18)

\[
q(t) = 1 + \Phi \left( \frac{\dot{k}(t)}{k(t)} + g + n + \delta \right)
\]  

(19)

\[
\left( r(t) + \delta - \frac{\dot{q}(t)}{q(t)} \right) q(t) = \alpha A k(t)^{-(1 - \alpha)} \frac{\Phi}{2} \left( \frac{\dot{k}(t)}{k(t)} + g + n + \delta \right)^2
\]  

(20)

where,

\[
k(t) = \frac{K(t)}{h(t)L(t)} = \frac{K(t)}{e^{\gamma t+n} t}
\]  

(21)

\( k \) is defined as capital per efficiency unit of labor.

3. The Adjustment Path and the Steady State under Financial Autarky

We define as \textit{financial autarky}, the regime under which the economy cannot borrow or lend internationally. Under financial autarky, equilibrium in the goods market requires that domestic consumption plus investment are continuously equal to total domestic output. Thus, financial autarky is a regime in which the economy behaves as a closed economy, and domestic investment is always equal to domestic savings.
The dynamic properties of the model under financial autarky are well known from the standard Ramsey-Cass-Koopmans model with adjustment costs for investment (see Abel and Blanchard 1983).

From the production function (13), output per efficiency unit of labor is given by,

\[ y(t) = Ak(t)^\alpha \]  \hspace{1cm} (22)

where, \( y(t) = \frac{Y(t)}{h(t)L(t)} = \frac{Y(t)}{e^{(\rho+n)t}} \).

From the aggregate consumption function (5), consumption per efficiency unit of labor evolves according to,

\[ c(t) = (r(t) - \rho - g)c(t) \]  \hspace{1cm} (23)

Under financial autarky, the economy must satisfy,

\[ y(t) = Ak(t)^\alpha = c(t) + q(t)\left( k(t) + (g + n + \delta)k(t) \right) \]  \hspace{1cm} (24)

(24) can be rewritten as,

\[ k(t) = \frac{1}{q(t)}\left( Ak(t)^\alpha - c(t) \right) - (g + n + \delta)k(t) \]  \hspace{1cm} (25)

Under the assumption that \( 0 < \alpha < 1 \), the model possesses a steady state.

We can use the model to analyze the balanced growth path under financial autarky as well as the adjustment process towards the balanced growth path.

3.1 The Balanced Growth Path

The balanced growth path (steady state) under autarky is defined as the vector \((y_E, k_E, c_E, q_E, r_E, w_E)\) that simultaneously satisfies (18), (19), (20), (22), (23) and (25), for,

\[ q(t) = k(t) = c(t) = 0 \]

The steady state turns out to be a function of a single state variable, namely \( k \).

From (23), in the steady state, the real interest rate must satisfy,

\[ r_E = \rho + g \]  \hspace{1cm} (26)

From (19), the steady state shadow price of installed capital must satisfy,
\[ q_E = 1 + \phi (g + n + \delta) \]  

Equation (27) determines the steady state investment rate.

From (20), the equality between the user cost of capital and the marginal product of capital, must hold for the steady state investment rate, the steady state real interest rate and the steady state capital stock. Substituting (26) in (20), this implies that,

\[ q_E = \frac{1}{\rho + g + \delta} \left( \alpha A k_E^{-(1-\alpha)} + \frac{\phi}{2} (g + n + \delta)^2 \right) \]  

(28)

(27) and (28) determine the steady state capital stock per efficiency unit of labor as,

\[ k_E = \left( \frac{a A}{\rho + g + \delta + (\phi(\rho - n) + (\phi/2)(g + n + \delta))(g + n + \delta)} \right)^{1/(1-\alpha)} \]  

(29)

Note from (29) that the steady state capital stock is lower than the steady state capital stock in the absence of adjustment costs for investment (\( \phi = 0 \)). Adjustment costs for gross investment result in a lower steady state capital stock.

Once the steady state capital stock per efficiency unit of labor is determined, steady state output follows from (22), the steady state real wage follows from (18) and steady state consumption per efficiency unit of labor follows from (25).

From (18), the real wage per efficiency unit of labor, \( \omega \), is given by,

\[ \omega_E = e^{-\gamma t} w_E = (1-\alpha) A k_E^{\alpha} \]  

(30)

Finally, from (25), steady state consumption per efficiency unit of labor satisfies,

\[ c_E = A k_E^{\alpha} - (g + n + \delta) q_E k_E \]  

(31)

3.2 Dynamic Adjustment to the Balanced Growth Path

The dynamic behavior of the full model is determined by the evolution of three variables. The capital stock \( k \), which is the only state (backward looking) variable in this model, \( q \), which determines the investment rate and is a control (forward looking) variable, and \( c \), private consumption, which is also a control variable. The behavior of the real wage \( \omega \) and the real interest rate \( r \) follow directly from the paths of these three variables.

The stability properties of this type of growth model have been studied formally by Abel and Blanchard (1983). The balanced growth path is a stable manifold and the adjustment process is governed by three roots, one negative (stable) which corresponds to the capital stock \( k \), and two positive (unstable), which correspond to the forward looking variables \( q \) and \( c \).
In what follows, we provide a heuristic description of the dynamic adjustment paths using a pair of interdependent phase diagrams, which help describe the economics of the adjustment process for investment, consumption and the capital stock. Without loss of generality we focus on the case of an economy with an initial capital stock which is lower than the steady state capital stock.

Assume that the economy in time $t_0$ possesses a capital stock per effective unit of labor which is smaller that the steady state capital stock, say $k_0 < k^E$. With a low initial capital stock, the real interest rate $r$, as well as $q$ (the investment rate) will be higher than on the balanced growth path. As the economy accumulates capital, both the real interest rate and the investment rate are declining along the adjustment path. Since along the adjustment path the real interest rate is higher than in the steady state, consumption per efficiency unit of labor will be rising as the economy is adding to its stock of capital per effective unit of labor. The growth rate will be higher than the steady state growth rate, and the economy will gradually converge to its balanced growth path. On the other hand, during the convergence process, $q$ and the investment rate will be on a declining path and consumption per effective unit of labor on a rising path. This adjustment process in depicted in Figure 1.

The dynamics of investment ($q$) and the capital stock ($k$) are qualitatively similar to the dynamics of the standard $q$ model with an exogenous real interest rate. However, in our case the real interest is endogenous, as it simultaneously satisfies (20) and (23). Thus, the adjustment path of $q$ would lie above the corresponding path in the constant real interest rate case (dotted line), as the real interest rate is declining along the adjustment path in this model.

The dynamics of consumption and the capital stock are qualitatively similar to the dynamics of the standard Ramsey-Cass-Koopmans model, without adjustment costs for investment. However, the adjustment path of consumption is steeper than the adjustment path of that model, as, outside the steady state, the path of real interest rates lies above the corresponding path in the absence of adjustment costs for investment.

In Figure 2 we depict the time paths of output and consumption in the case of financial autarky. Both start below their steady state values and gradually converge towards them. Consumption converges at a faster rate, as the investment rate is declining along the adjustment path. Initially, the real interest rate will be higher than the steady state real interest rate and the real wage rate lower that in the steady state, but growing at a rate higher than $g$. In the steady state the real interest rate will converge to $\rho + g$ (see eq. 26), while wages per effective unit of labor will be increasing, due to the accumulation of capital, which causes an increase in the marginal product of labor (see eq. 30).

It is straightforward to deduce from Figure 1 the nature of the dynamic adjustment of an economy whose initial capital stock is above its steady state value. With a high initial capital stock, the real interest rate $r$, as well as $q$ and the investment rate will be lower than in the steady state. As the economy gradually de-cumulates capital, the real interest rate and the investment rate are rising along the adjustment path. Since along the adjustment path the real interest rate is lower than in the steady state, consumption per efficiency unit of labor will be falling, as the economy is subtracting from its stock of capital per effective unit of labor. The growth rate will be lower than the steady state growth rate, and the economy will gradually converge to its balanced growth path.

We next turn to an analysis of the model under financial openness.
4. Financial Openness under Pre-Commitment in a Small Open Economy

We define financial openness as a regime in which the economy can borrow and lend freely at the world real interest, and is not therefore constrained to finance domestic investment through domestic savings.

We shall assume for simplicity that the world real interest rate is equal to the steady state real interest rate under autarky. Essentially, this means that the rest of world is already on a balanced growth path, that consumers in the rest of the world have the same rate of time preference as domestic consumers, and that the exogenous rate of increase of productivity is the same as in the rest of the world. Thus, the economy we are studying is not only “small” but also “poor” relative to the rest of the world, which is already on the balanced growth path.

We shall also assume that this economy is able to pre-commit not to default on its external debt.

Thus the world real interest rate at which this economy can borrow and lend internationally is given by,

\[ r^* = \rho + g \]  \hspace{1cm} (32)

4.1 Consumption under Financial Openness

Under financial openness, consumption per effective unit of labor will jump immediately to its relevant steady state level, as consumers will be able to fully smooth their consumption path.

Substituting (32) in the Euler equation for consumption (23), we get,

\[ c(t) = (r^* - \rho - g) c(t) = 0 \]  \hspace{1cm} (33)

Since households are assumed to satisfy their inter-temporal budget constraint, it must also follow from (4), that,

\[ k(0) + \int_{t=0}^{\infty} w(t) e^{-(r(s)-n)dt} \frac{dt}{t} = \int_{t=0}^{\infty} c(t) e^{-(\rho-g)dt} \frac{dt}{t} \]  \hspace{1cm} (34)

In (34) we have assumed that the initial wealth of the representative household is only in the form of domestic capital. Recall that \( w(t) \) is the real wage per worker and \( c(t) \) is consumption per effective unit of labor. With a constant real interest rate as in (32), (34) can be rewritten as,

\[ k(0) + \int_{t=0}^{\infty} w(t) e^{-(r(s)-n)dt} \frac{dt}{t} = \int_{t=0}^{\infty} c(t) e^{-(\rho-n)dt} \frac{dt}{t} = \int_{t=0}^{\infty} c(t) e^{-(\rho-n)dt} \frac{dt}{t} = \frac{c(k_0)}{\rho - n} \]  \hspace{1cm} (35)

This assumption simplifies the analysis for a “small” open economy, but involves little loss of generality. To confirm this, see our two country analysis below, where the world real interest rate is allowed to be different from its steady state value.
where \( \bar{c} \) denotes the constant (smoothed) consumption per effective unit of labor. Solving (35) for consumption, we get,

\[
\bar{c}(k_0) = (\rho - n) \left( k(0) + \int_{t=0}^{\infty} w(t) e^{-(\rho+g-n)t} \, dt \right)
\]  

(36)

Equation (36) suggests that under financial openness, and with the world real interest rate at its steady state value, domestic consumption per effective unit of labor will be determined as postulated by the permanent income hypothesis. Domestic consumers will be consuming their permanent income, which is a constant fraction of their total wealth.\(^6\)

Note that consumption per effective unit of labor will be constant and a positive function of the initial capital stock \( k_0 \). The higher is the initial capital stock, the higher will be the permanent income of the representative household, since both its original non-human wealth \( (k_0) \) and its human wealth will be higher. Its human wealth will be higher because, with a higher initial capital stock, the path of real wages will also be higher, and thus the present value of the stream of labor income in (35) will be higher when evaluated at the world real interest rate.

Also note that with \( k_0 < k_E \) savings under openness will be lower than savings under autarky, since consumption will be equal to permanent income and not current (low) income minus investment.

4.2 Investment under Financial Openness

With the world real interest constant at its steady state value, investment will be determined by (19) and the equality between the user cost of capital and the marginal product of capital.

\[
\left( r^* + \delta - \frac{\dot{q}(t)}{q(t)} \right) q(t) = \left( \rho + g + \delta - \frac{\dot{q}(t)}{q(t)} \right) q(t) = \alpha A k(t)^{1-\alpha} + \frac{\phi}{2} \left( \frac{\dot{k}(t)}{k(t)} + g + n + \delta \right)^2
\]

(20′)

Whereas in the steady state (20) and (20′) will coincide, (20′) will lie above (20) for economies with an initial capital stock below the steady state capital stock, and below (20) in the opposite case. This means, that an economy with a low initial capital stock will experience higher investment under financial openness than under autarky, and will as a result converge more rapidly to the steady state. This is because world interest rates are lower compared to this economy’s initial autarky interest rate. An economy with a high initial capital stock, i.e one that exceeds its steady state level, will also experience faster convergence, as it will have a higher disinvestment rate, since the world interest rate will be higher than its initial autarky rate.

The difference between autarky and openness is depicted in Figure 3. The saddle path under openness is steeper than under autarky (the dotted line), and as a result convergence is faster. On the other hand, both types of the adjustment path lead to the same steady state capital stock and domestic output.

\(^{\text{Note, however, that although consumption per effective unit of labor will be constant, consumption per head will be rising at a constant rate } g.\)
We have thus demonstrated that, for an economy with a low initial capital stock relative to its steady state value and the rest of the world, savings are lower and investment is higher under financial openness, than in the case of autarky. As a result, after financial liberalization, the current account will initially move into deficit.

4.3 The Current Account and External Balance

The evolution of the current account will be determined by the difference between national savings and domestic investment. Thus, instead of (25), which in the case of autarky required the equality of domestic savings and investment, we shall have,

$$f(t) = \left( (r^* - g - n)f(t) + Ak(t)^a - c(t) \right) - q(t) \left( k(t) + (g + n + \delta)k(t) \right)$$

(25′)

where \( f \) denotes net financial assets from the rest of world. To the extent that national savings exceed domestic investment, the economy has a current account surplus, and accumulates positive net financial assets from the rest of the world. In the opposite case, it has a current account deficit, and accumulates negative net financial assets, i.e. net foreign debt.\(^7\)

The trade balance, will be determined by the difference between domestic savings and investment, and will be given by,

$$Ak(t)^a - c(t) - q(t) \left( k(t) + (g + n + \delta)k(t) \right)$$

We shall again concentrate on a small economy that starts with a low initial capital stock and will additionally assume that net financial assets from the rest of the world are initially zero.

We have already demonstrated that under financial openness this economy will initially have lower savings and higher investment compared to the case of autarky. Thus, its trade balance and the current account will move into deficit and the economy it will start accumulating foreign debt. Substituting (31) and (33) into (25′), the process of debt accumulation will be determined by,

$$f(t) = (\rho - n)f(t) + Ak(t)^a - \tilde{c}(k_0) - q(t) \left( k(t) + (g + n + \delta)k(t) \right)$$

(36)

As the economy converges towards the steady state, national savings increase, since national income increases and consumption is constant, while domestic investment gradually falls, as \( q \) falls due to the gradual increase of the stock of physical capital. In the steady state, national savings (per effective unit of labor) will be equal to domestic investment, and the stock of equilibrium net foreign assets per effective unit of labor will be negative, because of the accumulation of current account deficits during the adjustment process. Steady state net foreign assets will be given by,

\(^7\) Those net financial assets \( f \) could in principle take the form of either bonds (debt) or shares (equity). These are perfect substitutes in this model, as there is no uncertainty. Thus, current account deficits can be financed either through international borrowing or foreign direct and portfolio investment. When we discuss the possibility of default, default must thus be interpreted widely, as both debt repudiation and nationalization of foreign held equity in domestic firms.
Note from (37) that steady state domestic output per effective unit of labor is the same as under autarky. The same applies to $q$, the real wage and the capital stock. Steady state (replacement) investment is thus the same as under autarky. The reason is that under autarky the steady state domestic real interest rate is equal to the world real interest rate.

In Figure 4, we present the adjustment process. Note that we consider a small economy, whose initial capital stock is below its steady state value, under the assumption that the rest of the world is on a balanced growth path. For this economy, the path of real interest rates under financial openness will be below the corresponding path under autarky. As a result, both per capita consumption and investment will be higher during the adjustment process under financial openness than under autarky. Lower interest rates and higher investment imply higher real wages and a higher present value of labor income under openness than under autarky. Thus, both the present value of consumption and the welfare of the representative household will be higher under financial openness than under autarky.

During the transition, the small open economy in question runs current account deficits and accumulates foreign debt. As the economy approaches the steady state, the process of foreign debt accumulation slows, and the economy returns to external balance. However, steady state consumption per capita is lower under financial openness than under autarky, as under openness, the economy has to pay interest on the foreign debt that it has accumulated during the transition.

Consider Figure 4. At time $0$, because of the low initial capital stock, consumption is higher than output minus investment. Thus, there is a trade deficit, which is equal to the difference between the high consumption and the low output net of domestic investment. As output gradually rises and investment gradually declines along the adjustment path, the trade deficit narrows and after a point becomes a surplus. From then on the economy is running trade surpluses in order to service the foreign debt that it has accumulated. In steady state the economy returns to external balance, servicing a constant foreign debt per effective unit of labor.

Although steady state domestic output (GDP) per capita is the same under financial autarky and openness, national income (GNP) per capita is smaller than domestic output (GDP) per capita under openness, as the country has to pay interest on the debt it has accumulated vis-a-vis the rest of the world.

It is also worth noting that on the balanced growth path the economy runs a trade surplus, but a current account deficit. Foreign debt per effective unit of labor is constant, meaning that foreign debt is rising at a rate $g+n$, the same as the rate of growth of domestic output (GDP).

For an economy with an initial capital stock that exceeds the steady state capital stock, the opposite would apply. Under financial openness it will initially experience trade and current account surpluses, and in the steady state it will end up with positive net foreign assets rather than foreign debt. Consumption per effective unit of labor will be higher than under autarky in the steady state, because the country receives interest payments on the foreign assets it has accumulated. Although steady state domestic output (GDP) per capita is the same under financial autarky and openness,
national income (GNP) per capita is higher than domestic output (GDP) per capita under openness, as the country receives interest on the assets it has accumulated vis-a-vis the rest of the world.

The dependence of steady state consumption on the initial capital stock under openness can be directly deduced from (35). The higher the initial capital stock, the higher will be the permanent income of the representative household, since both its original non-human wealth \((k_0)\) and its human wealth will be higher. Its human wealth will be higher because with a higher initial capital stock, the path of real wages will be higher, and thus the present value of the stream of labor income in (35) will be higher when evaluated at the world real interest rate.

5. Financial Openness under Pre-Commitment in a Two-Country World

We now abandon the small open economy assumption, in order to analyze the case of a two country world of interdependent “poor” and “rich” economies. We again analyze the equilibrium paths under the assumption that the “poor” economy can pre-commit not to default on its international debt.

Assume a world economy consisting of two economies that are similar in every respect, apart from their initial per capita capital stocks. Both economies are competitive, they have access to the same production technology, and their consumers have exactly the same preferences. We shall assume that economy 1 has a higher initial capital stock than economy 2, relative to its steady state value. Thus, economy 1 is the relatively “rich” economy and economy 2 the relatively “poor” economy.

The two country world is described by the following model, where subscript \(i=1,2\) refers to the two countries.

\[
y_i(t) = A k_i(t)^a
\]

\[
c_i(t) = (r(t) - \rho - g) c_i(t)
\]

\[
q_i(t) = 1 + \phi \left( \frac{k_i(t)}{k_i(t)} + g + n + \delta \right)
\]

\[
\left( r_i(t) + \delta - \frac{q_i(t)}{q_i(t)} \right) q_i(t) = \alpha A k_i(t)^{-(1-\alpha)} + \phi \left( \frac{k_i(t)}{k_i(t)} + g + n + \delta \right)^2
\]

\[
\omega_i(t) = w_i(t) e^{-\gamma t} = (1 - \alpha) A k_i(t)^a
\]

\[
 f_i(t) = \left( (r(t) - g - n) f_i(t) + Ak_i(t)^a - c_i(t) \right) - q_i(t) \left( k_i(t) + (g + n + \delta) k_i(t) \right)
\]

\[
 f_i(t) = -f_i(t)
\]

\[
k_E > k_1(0) > k_2(0)
\]
Under financial autarky, net foreign assets $f$ are assumed equal to zero for all $t$. Under financial openness, the two economies have the same real interest rate $r(t)$, the world real interest rate, which is determined by the equality of savings and investment in the world economy. It is straightforward to show that the world real interest rate will lie between the autarky real interest rates of the two economies. It will be higher than the autarky real interest rate for economy 1 (the “rich” economy) and lower than the autarky real interest rate for economy 2 (the “poor” economy). Thus, economy 1 will have lower investment and consumption (higher savings) than in the case of autarky, and economy 2 will have higher investment and consumption (lower savings) than in the case of autarky.

It follows from our previous analysis of a single economy, that in the transition path economy 1 will have a lower growth rate of output per capita than in the case of autarky, while economy 2 will have a higher growth rate. More importantly, in the transition path, economy 1 will be accumulating net foreign assets, through current account surpluses, while economy 2 will be accumulating foreign debt, through current account deficits.

The transition paths for investment and the capital stock for the two economies are depicted in Figure 5, under the additional assumption that the initial capital stock of both economies is below its steady state equilibrium. Because the path of the world real interest rate will lie between the paths of the autarky real interest rates in the two economies, the path of investment will be lower than under autarky for economy 1 (the capital rich economy) and higher than under autarky for economy 2 (the capital poor economy), and the opposite will happen for the path of savings. However, both economies will be converging towards the same balanced growth path for capital and output.

Where things differ qualitatively from our previous analysis is in the paths of private consumption and the current account. Unlike the small open economy case that we analyzed in the previous section, the world real interest rate is above its steady state value $\rho + g$, and it is not constant but declining along the transition path. Thus, consumption smoothing will not be absolute, as in the case of a constant equilibrium real interest rate, but relative.

As depicted in Figure 6, consumption per efficiency unit of labor will be increasing in both economies, as they accumulate capital and the world real interest rate is higher than its steady state, and on a declining path towards its steady state value. In economy 2 (the “poor” economy), consumption will be initially above the difference between output and investment, and as a result economy 2 will be experiencing current account deficits. The reason is that the world real interest rate is lower than its “autarky” real interest rate. In economy 1 (the “rich” economy), consumption will be below the difference between output and investment, and economy 1 will be experiencing current account surpluses. The reason is that the world real interest rate is higher than its “autarky” real interest rate. As consumption and output are rising in both economies, there will be a gradual narrowing of the trade imbalances, which after some time will be reversed. Economy 2 will start experiencing trade (but not current account) surpluses, in order to service the foreign debt that it has accumulated, while economy 1 will start experiencing trade (but not current account) deficits. As the economies converge towards the balanced growth path, external balance is restored, in the sense that net foreign assets per efficiency unit of labor are stabilized for both economies.

In the steady state, both economies will have converged to the same domestic output (GDP) per effective unit of labor, but steady state consumption (and GNP) will be higher in economy 1, which
is a net lender vis-a-vis the rest of the world, and lower in economy 2, which is a net borrower from
the rest of the world (economy 1).

We have thus demonstrated the following five propositions:

1. On the balanced growth path, domestic output (GDP) per capita under financial openness is the
same as under autarky for both economies.
2. On the balanced growth path national income (GNP) and consumption per capita are lower than
in the case of financial autarky for economy 2 (the “poor” economy), since the economy has to
pay interest on the foreign debt it has accumulated during the transition. The opposite applies to
economy 1 (the “rich” economy).
3. During the transition, the “rich” economy runs current account surpluses and accumulates net
foreign assets, while the “poor” economy runs current account deficits and accumulates net
foreign debt.
4. The balanced growth path is characterized by external balance, in the sense of constant net
foreign assets (per effective unit of labor), for both the “rich” and the “poor” economy. The
“rich” economy is a net international creditor, and the “poor” economy a net international debtor.
5. There are benefits from inter-temporal trade for both types of economies, as, during the transition
path, the real interest rate under autarky differs from the world real interest rate for either of
them.
6. The benefits of financial openness are “front loaded” for the “poor” economy and the costs are
“back loaded”. The opposite is the case for the “rich” economy. Both consumption and
investment rise initially relative to autarky in the “poor” economy, resulting in a period of trade
deficits, which has to be followed by a period of trade surpluses. In the rich economy, the
opposite happens.

In the Appendix we present numerical simulations of both the small open economy and two country
models. This allows us to get a quantitative feel of the significance of the differences between
financial openness and autarky in the presence of adjustment costs for investment. The inter-
temporal benefits from financial openness are quite small quantitatively, although the inter-temporal
tradeoffs involved are quite sharp.

6. The Time Inconsistency of International Borrowing

Our analysis of the augmented neoclassical optimal growth model, has demonstrated that financial
openness affords both “poor” and “rich” economies the opportunity to engage in beneficial inter-
temporal trade, as long as the path of the world real interest rate differs from the path of real interest
rates under autarky and there is pre-commitment. We have demonstrated that this will be the case
during the adjustment path to a balanced growth path, as long as the initial capital stock of an
economy differs from the initial capital stock in the rest of the world, even if the economy is
caracterized by the same technology and preferences as the rest of the world.

However, the predictions of this augmented optimal growth model are at odds with two important
stylized facts. The very high correlation between domestic savings and investment rates in open
economies, first highlighted by Feldstein and Horioka (1980), and the fact that capital does not flow
from “rich” to “poor” countries as predicted by the model, a paradox noted and analyzed by Lucas
These stylized facts constitute puzzles and paradoxes for the predictions of the optimal growth model under the assumption of financial openness and pre-commitment.\(^8\)

In this section we provide an explanation of the failure of the above analysis of the optimal growth model to account for these facts, it terms of the time inconsistency of the optimal policy on the part of “poor” economies, i.e. international borrowers, and not necessarily on other features of the optimal growth model.\(^9\)

### 6.1 Lack of Pre-commitment and the Time Inconsistency of International Borrowing

The optimal policy we have analyzed, assuming there is pre-commitment, is time inconsistent for a “poor” economy. It is time inconsistent because the benefits arise in the first part of the adjustment process, and the costs are paid in the latter part of the process. The implementation of the optimal policy requires ex ante pre-commitment on the part of the “poor” economy, not to re-optimize at any time in the future. Formally, the loan contract must be a legally binding infinite horizon contract, signed when the “poor” economy enters international capital markets, that precludes any future change in the terms of the contract, let alone default.

Although such contracts are legally enforceable in national jurisdictions, international borrowing by sovereign states is different. The law may deter a private borrower in a national jurisdiction from defaulting, but nations have found it easier to default, as it is more difficult to apply legal sanctions to entire nations. Thus, when nations have incentives to default rather than continue honoring their debts, they will do so. To a large extent, financial openness operates on a “willingness to pay” basis, as contracts with foreign sovereigns are not as easily enforceable as contracts with private citizens within a national jurisdiction.\(^10\)

As Eaton, Gersovitz and Stiglitz (1986) have commented, “One problem in all lending is enforcement, the difficulty in ensuring that both sides of a contract adhere to its terms. Here, the particular concern is the difficulty of ensuring that the borrower pays the lender. The major difference between domestic and international debt is that the former are legal obligations, enforceable in courts. Another difference is that, domestically, debtors who cannot meet their obligations have the option of filing for bankruptcy. Repayment of international debt, however, is largely voluntary; the penalties to be imposed on a country that does not honor a contract are, at best, indirect. On the other hand, there is no systematic procedure, corresponding to bankruptcy, by which a country that has undertaken an excessive amount of debt can discharge its obligations and proceed on its way.” p.484 (italics added).

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\(^8\) See Obstfeld and Rogoff (2000), for an account of the major puzzles in international macroeconomics.

\(^9\) See Kydland and Prescott (1977) and Calvo (1978) for the early pioneering analyses of the time inconsistency problem in dynamic settings. In the context of international borrowing, the time inconsistency problem was first studied in the important contribution of Eaton and Gersovitz (1981), Cohen and Sachs (1986), Bulow and Rogoff (1989) and Cohen (1991) among others have also contributed to this literature. An important recent application, that also surveys the more recent literature, is Arellano (2008). However, the models used in this literature are much simpler than the augmented optimal growth model of this paper. They are either two period Fisherian models with capital, or endowment economies without capital. An exception is Cohen and Sachs (1986) who use a linear production technology.

\(^10\) See Eichengreen and Portes (1986), Eichengreen and Lindert (1989) and Reinhart and Rogoff (2009) for historical accounts of sovereign default episodes and the complex negotiations that ensued.
In the absence of sufficient pre-commitment mechanisms, an international borrower will, after a point, find it beneficial to default on the accumulated foreign debt, even if this means returning to financial autarky. In fact, defaulting on foreign debt and returning to financial autarky becomes the optimal regime in our model once the borrower has to run trade surpluses in order to service the debt. A default will allow an international borrower to increase consumption above the level that can be attained by sticking to the ex ante optimal policy. Thus, the optimal policy is time inconsistent and international borrowers would suffer from lack of credibility.

The incentives of international borrowers to default will be anticipated by international lenders who will adapt their behavior so that time inconsistency does not arise. The time consistent equilibrium may imply that financial openness completely breaks down, or that there are significant default risk premia on interest rates charged to international borrowers, or even lending limits and sanctions against defaulting international borrowers. In the remainder of this section we first analyze the time inconsistency problem and then discuss alternative solutions that ensure time consistency.

In order to analyze the time inconsistency problem we must derive the value function of the representative household in each country. Under financial openness, this measures the maximum welfare of the representative household as a function of the two state variables \( k \) and \( f \). Under financial autarky \( f \) is equal to zero.\footnote{Since the optimal growth model is deterministic, we stick to a deterministic framework. In such a framework only “willingness to pay” matters, as issues such as moral hazard or adverse selection, that would arise in a model with extraneous uncertainty and asymmetric information do not arise. See Eaton, Gersovitz and Stiglitz (1986) on this point.}

On the balanced growth path, the value function per efficiency unit of labor is defined by,

\[
V(k_E, f_E) = \int_{s=0}^{\infty} e^{-(\rho-n)s} \ln c_k ds = \frac{\ln c_E}{\rho - n} \tag{39}
\]

where subscript \( E \) denotes steady state values.

From the national income identity (38.6), under financial openness, optimal steady state consumption is given by,

\[
c_E = (\rho - n) f_E + A(k_E)^\alpha - (n + g + \delta) q_E k_E \tag{40}
\]

where, \( q_E = 1 + \phi(n + g + \delta) \).

Substituting (40) in (39), the steady state value function takes the form,

\[
V(k_E, f_E) = \frac{1}{\rho - n} \ln \left( (\rho - n) f_E + A(k_E)^\alpha - (n + g + \delta) q_E k_E \right) \tag{41}
\]

(41) depends only on the stocks of net foreign assets and domestic capital, and the exogenous parameters of the model. Since replacement investment is lower than domestic output, the steady state value function is a positive function of both steady state net foreign assets and steady state domestic capital.
Under financial autarky, \( f_E = 0 \). Under financial openness, for a “poor” economy \( f_E < 0 \), and for a “rich” economy \( f_E > 0 \). Thus, from (41), compared to financial autarky, a “poor” economy is worse off in the steady state under financial openness than under autarky, while a “rich” economy is better off. Thus, the steady state value function under financial autarky is given by,

\[
V'(k_E) = V(k_E, f_E = 0) = \frac{1}{\rho - n} \ln \left( A(k_E)^{\alpha} - (n + g + \delta)q_E k_E \right)
\]  

(42)

(42) is clearly higher than (41) for a “poor” debtor nation. Clearly, in the steady state, such a nation would always find it optimal to “default” on its foreign debt and return to autarky, as this would increase its welfare to the higher level of welfare under autarky. From (41) and (42),

\[
V(k_E, f_E) < V'(k_E) \text{ for } f_E < 0
\]

(43)

Thus, ex post default is always optimal in the case of “poor” economy when it has arrived at the steady state.

One can prove that default will become optimal before the steady state is approached.

Recall that initially, at time 0, the value function of a “poor” debtor nation is higher under financial openness than under autarky, and that consumption is higher under financial openness than under autarky, for as long as the economy is characterized by trade deficits (see Figure 6). Thus, the value function under financial openness starts from a higher initial value relative to autarky, and approaches a lower steady state value than under financial autarky. Thus, the two value functions will cross before the steady state is achieved.

Figure 7 depicts the path of the two value functions from time zero to infinity. Starting at time 0, the financial openness value function is greater than the autarky value function. Initially, the gap between the two remains positive, as the economy is growing faster under financial openness. At some point, because of the accumulation of external debt, the rate of increase in the value function under financial openness starts diminishing relative to the rate of increase of the value function under autarky. The gap starts to close, as the change in the value function under financial openness starts slowing down more rapidly than the change in the value function under financial autarky. At some point the two value functions will cross. At this point default becomes optimal for the debtor nation.

In the neighborhood of the steady state, the value function at time \( t \) under financial openness can be approximated as a linear function of the form,

\[
V(k(t), f(t)) = V(k_E, f_E) + V_k(k(t) - k_E) + V_f(f(t) - f_E)
\]

(44)

where,

\[
V_k = \frac{\partial V(k_E, f_E)}{\partial k_E} = \frac{\alpha A(k_E)^{\alpha - 1} - (n + g + \delta)q_E}{(\rho - n)(\rho - n) f_E + A(k_E)^\alpha - (n + g + \delta)q_E k_E} > 0
\]

(45)
\[ V_f = \frac{\partial V(k_E,f_E)}{\partial f_E} = \frac{1}{(\rho - n)f_E + A(k_E)^\alpha - (n + g + \delta)q_E k_E} > 0 \]  

(46)

For \(0 < k(t) < k_E\), \(V_f(k(t)-k_E)\) is negative, but converges towards zero at a speed \(V_f\). Thus, the contribution of capital accumulation to the value function is rising as the steady state is approached. For \(0 > f(t) > f_E\), as is the case in a “poor” debtor nation, \(V_f(f(t)-f_E)\) is positive and converges to zero at a speed \(V_f\). Thus, the contribution of foreign debt accumulation to the value function is falling as the steady state is approached.

From (42), in the neighborhood of the steady state, the financial autarky value function at time \(t\), is equal to,

\[ V'(k(t)) = V'(k_E) + V_k'(k(t) - k_E) \]  

(47)

where,

\[ V_k' = \frac{\partial V'(k_E)}{\partial k_E} = \frac{\alpha A(k_E)^{\alpha-1} - (n + g + \delta)q_E}{(\rho - n)(A(k_E)^\alpha - (n + g + \delta)q_E k_E)} \]  

(48)

For \(k(t) < k_E\), the value function under financial autarky is lower than the steady state value function, but rises towards it at a speed \(V_k'\).

By comparing (48) with (45), we can confirm that,

\[ 0 < V_k' < V_k \]  

(49)

From (49), deviations of the current capital stock from its steady state value have a smaller impact on the current financial autarky value function than the current financial openness value function. However, the speed of adjustment of the current financial openness value function is also affected by the accumulation of foreign debt, which slows down its increase, as shown in the analysis of the differential equation (44).

As long as the value function under financial openness is higher than the value function under financial autarky, the country will not choose to default. It will only choose to default at the instant when,

\[ V'(k(t)) - V(k(t),f(t)) \geq 0 \]  

(50)

(50) measures the net value of a default at time \(t\). The moment that this becomes positive, i.e the moment that the current value function in the case of financial autarky exceeds the current value function in the case of financial openness, a country will default.

From (44) and (47), (50) implies that this will happen at the instant when,

\[ V'(k_E) - V'(k_E,f_E) - (V_k - V_k')(k(t) - k_E) - V_f(f(t) - f_E) \geq 0 \]  

(51)
can be rearranged as,

\[(f(t) - f_E) \left( V_f + \left( V_k - V_k' \right) \frac{k(t) - k_E}{f(t) - f_E} \right) \leq V'(k_E) - V(k_E, f_E) > 0\]  

(52)

It is straightforward to prove that the left hand side of (52) tends to zero as foreign debt and capital converge towards their steady state values. Since the steady state of the value of defaulting minus the value of not defaulting on the right hand side is positive, a default will occur before foreign debt and capital achieve their steady state values. Thus, a default will take place before the steady state in reached.

Recalling our analysis of the path of consumption in Figure 6, this will happen at the moment when the initial trade deficits are transformed into trade surpluses. By defaulting on its international debt, the poor economy can start increasing domestic consumption and the welfare of the representative household faster, instead of running trade surpluses to service its foreign debt. This is how the time inconsistency problem arises. As long as (50) is not satisfied, which happens as long the “poor” economy is running trade deficits, it has no incentive to default. As soon as (50) is satisfied, there is an incentive to renege on its previous commitments and default.\(^{12}\)

The incentives of a “poor” economy to default will be anticipated by international lenders. Therefore, in the case where there cannot be pre-commitment to the ex ante optimal path, as we have assumed in the analysis of the previous sections, a “poor” economy is anticipated to default on its debts once condition (50) is satisfied. Financial openness will then break down, as in the time consistent equilibrium, even without any legal restrictions on international capital flows, there will be no international borrowing or lending. Lenders will not lend at the world real interest rate, in the knowledge that borrowers will eventually default.

Thus, under lack of pre-commitment, capital will not flow from “rich” to “poor” countries. Both the Feldstein Horioka puzzle and the Lucas paradox can thus be explained. The time consistent equilibrium under financial openness will look exactly like financial autarky, despite the absence of controls on capital flows.

6.2 Pre-commitment Mechanisms and Sanctions as Solutions of the Time Inconsistency Problem

In repeated games, a potential solution to the time inconsistency problem is the acquisition of a reputation for not succumbing to the incentives of following the ex post optimal policy. This solution cannot apply in this case, as the “convergence game” analyzed in this paper is not a repeated game. Although dynamic, it is only played once. An economy starts “poor” and converges to its steady state by accumulating physical capital. The fact that it services its international debt while continuing to borrow from abroad in the first stage of the game does not affect its reputation for not defaulting, as it is not yet in its national interest to default. By solving the equilibrium backwards, forward looking international lenders can find out that when the crunch comes, and condition (50) is satisfied, the “poor” country will have a irresistible incentive to default.

\(^{12}\) We assume here that the decision to default or not is taken centrally by the “government” of the “poor” economy, whose only function is to coordinate the decisions of domestic households and firms.
In business cycle games, such as those analyzed in a large part of the debt default literature mentioned above, where international borrowing and lending serve to finance temporary current account imbalances due to income fluctuations, one can contemplate a reputational solution to the credibility problem, as the game is repeated in every cycle. Not in the one shot “convergence game” of this paper.

A number of alternative solutions may work though. Conditional lending through international institutions like the World Bank and other international development banks, sanctions in the case of default, and strong pre-commitment mechanisms may help break the deadlock. We shall examine such mechanisms starting with pre-commitment mechanisms.

*Pre-commitment mechanisms* can take a direct form, such as a binding constitutional restrictions on nationalizations or default of foreign debt, binding clauses in international debt contracts that refer disputes to the jurisdiction of courts in the lender countries, or the signing of binding international treaties or loan agreements with international economic organizations to which debtor nations belong, such as the World Bank, the IMF or the OECD. Such agreements could imply pre-commitment not to default, conditionality and continuous monitoring. Pre-commitment mechanisms can also take an indirect form, through participation in a broader economic area, such as a free trade area, a customs union, or a single market, like the European Union, with rules that effectively preclude sovereign default.

The frequency of international debt crises in the latest era of financial openness since the 1970s, such as the Latin American, Asian and Euro Area crises, suggests that such pre-commitment mechanisms are at best imperfect, and that time consistency problems may undermine financial openness and the concomitant benefits of inter-temporal trade, even in the presence of such mechanisms.

One recent case in point is the experience of the economies in the periphery of the euro area (Greece, Portugal, Spain and Ireland). These economies opted for full financial openness when they entered the euro area in the late 1990s. Their adoption of the euro was interpreted as a non default clause. As a result, for almost ten years they experienced lower real interest rates, higher investment and growth rates and lower national savings rates. Their current account deficits soared and they were the first to be hit in the international financial crisis of 2008. They have since returned to high real interest rates and some were excluded from international financial markets, effectively returning to a regime of financial autarky.

A second, and older, solution to the credibility problem is the possibility of *sanctions* in the case of default. Sanctions can indeed take many forms. From the “gunboat” diplomacy of the 19th century, to the trade and other economic sanctions of the 20th and 21st century.

As we have already seen, the sanction of just excluding a country from future participation in the international capital market would not work in the context of the model in this paper. A “poor” country would only default in the case where it has converged sufficiently, and it no longer needs the international capital market. What if the return to financial autarky entailed additional costs through other types of sanctions imposed by international creditors?
Let us consider the steady state value function (41) of an international borrower. This is lower in the case of non default than in the case of default, because steady state net foreign assets are negative. It thus follows from (41) and (42), that,

$$V(k_E, f_E) = \frac{1}{\rho - n} \ln \left( (\rho - n) f_E + A(k_E)^{\alpha} - (n + g + \delta) q_f k_E \right) < V'(k_E) = \frac{1}{\rho - n} \ln \left( A(k_e)^{\alpha} - (n + g + \delta) q_f k_E \right)$$

Default pays in the steady state.

What if international creditors could impose permanent sanctions with an instantaneous steady state penalty $s_E$ in the case of default. Then, the steady state value function of a net debtor in the case of default would take the form,

$$V'(k_E, s_E) = \frac{1}{\rho - n} \ln \left( -s_E + A(k_e)^{\alpha} - (n + g + \delta) q_f k_E \right)$$

Comparing (50) with (41), one can see that default would be averted with certainty if,

$$s_E > (\rho - n) f_E$$

(51) defines optimal sanctions. If sanctions were to cost a defaulting nation more than the savings of the interest payments on its steady state international debt, it would choose not to default either in the steady state or before. The threat of sanctions would be sufficient to sustain the financial openness equilibrium, as it fully addresses the time inconsistency problem, and sanctions never need to be used. The threat of sanctions would make the optimal path time consistent for international borrowers.

Consider Figure 8, which is modified from Figure 7. Now reverting to autarky once a country has a positive foreign debt, would immediately imply the sanction (51). The path of the value function under financial autarky and sanctions is depicted as the curve approaching the value function under financial openness, as default cannot pay more than this in the steady state. The path of the value function under financial autarky and sanctions always lies below the path of the value function under financial openness. The two lines never cross. Thus, the time to consider default never comes. Financial openness becomes time consistent, and capital flows freely from “rich” to “poor” countries. The analysis of the previous sections becomes relevant, as the threat of sanctions equal to (51) is sufficient to solve the time inconsistency problem.

However, optimal sanctions, such as those implied by (51), may be quite large and infeasible. For example, in the case of our simulations in the Appendix, for the sanctions to satisfy (51) they must be between 2-5% of the debtor nation’s GDP, with a present value of between two (2) and (5) times of a nation’s GDP. What if sanctions of this magnitude are not feasible? Then the threat of imposing the optimal sanctions would not be credible.

Let us assume that the maximum feasible, and thus credible, sanction is equal to,

$$\bar{s} < (\rho - n) f_E$$

(52)
Sanctions would work in this case, but only in conjunction with capital market restrictions. One such restriction is a foreign debt ceiling. A debtor country would never default if the maximum amount it was allowed to raise in international capital markets satisfied,

\[ \tilde{f} < \frac{s}{\rho - n} \]  

(53)

In such a case, a “poor” country would be able to borrow in international capital markets, at market determined interest rates, but only up to the point where its foreign debt reached the level implied by (53). Thus, limits on the use of sanctions would imply foreign debt ceilings in this model. As long as foreign debt ceilings are binding, sanctions would never need to be used. Capital would in this case flow from “rich” to “poor” countries, but up to the point determined by the debt ceiling (53).

We have thus demonstrated the following four additional propositions:

1. The optimal borrowing policy of a “poor” nation is time inconsistent. Once the benefits of financial openness have been reaped, the optimal policy becomes defaulting on the accumulated foreign debt and returning to financial autarky.
2. The incentives of international borrowers to default will be anticipated by international lenders, who will not lend to “poor” nations, even if there are no legal restrictions to international borrowing and lending. In the absence of a solution to the time inconsistency problem of international borrowers, financial autarky is the only time consistent equilibrium. In such a case, capital would not flow from rich to poor countries, and the Feldstein Horioka puzzle and the Lucas paradox are fully explained.
3. Full solutions to the time inconsistency problem of international borrowers require devising effective pre-commitment mechanisms that preclude default, or sufficiently strong sanctions, the threat of which is sufficient to stop a “poor” nation from defaulting.
4. If sufficiently strong sanctions are not feasible, limited sanctions combined with foreign debt ceilings provide a partial solution to the time inconsistency problem. Capital would flow from “rich” to “poor” countries, but up to a foreign debt ceiling determined by the maximum feasible severity of sanctions.

6.3 Potential Extensions of the Model

The model could be extended to introduce other government functions and policies. Up to now we have not considered the role of the government, beyond its ability to take the decision to default or not on a country’s international debt.

Introducing a government that finances a path of government consumption through lump sum taxes, debt or money would not affect the conclusions in this representative household model, as the model is characterized by both Ricardian equivalence and the super neutrality of money (Barro 1974, Sidrauski 1967). The path of government consumption will affect private consumption, but not the investment and growth rates, or total domestic savings, which jointly determine the path of the current account. In addition, the choice between government debt and lump taxes will not have any real effects. The same would be true of introducing money and monetary growth. The only real variable that would be affected by the rate of growth of the money supply would be the demand for
real money balances. On the other hand, the investigation of the impact of distortionary taxes under the two regimes of financial autarky and openness would be an interesting avenue for future research.

Another avenue for future research would be to extend the analysis to the case of a neoclassical growth model with overlapping generations (Diamond (1965), Blanchard (1985), Weil (1989)). In such a model, government consumption, debt and money would have real effects, as Ricardian equivalence would not hold.\textsuperscript{13}

7. Conclusions

In this paper we have examined financial openness and the dynamics of the current account and international borrowing in a neoclassical growth model, augmented with adjustment costs for investment. We have focused on the issue of the time inconsistency of international borrowing, to demonstrate that if sufficient pre-commitment mechanisms or credible sanctions are not available, financial openness may look like exactly like financial autarky.

We have analyzed the relation between growth and the current account both on the balanced growth path and in the process of adjustment towards the balanced growth path under both financial autarky and openness.

For a “poor” economy, whose initial capital stock is further from the steady state than is the case in the rest of the world, the path of real interest rates under financial openness will be below the corresponding path of real interest rates under financial autarky. As a result, under financial openness and pre-commitment not to default, both per capita consumption and investment will be higher during the adjustment process. During the transition to the balanced growth path, such an economy thus runs current account deficits and accumulates foreign debt. As it approaches the balanced growth path, the process of foreign debt accumulation slows down, and the economy approaches a position of external balance.

On the balanced growth path, domestic output (GDP) per capita is the same as under autarky, but national income (GNP) and consumption per capita are lower under financial openness than under autarky, since the economy has to pay interest on the foreign debt it has accumulated during the transition.

The opposite applies to a “rich” economy, whose initial capital stock is nearer to the steady state than the rest of the world. During the transition, the economy runs current account surpluses and accumulates net foreign assets. Steady state consumption per capita will be higher under financial openness than under autarky, as the economy receives interest on the foreign assets that it has accumulated during the transition.

Financial openness is welfare improving for all countries, but implies inter-temporal tradeoffs. These inter-temporal tradeoffs imply time inconsistency for regimes of financial openness, as “poor” countries will reach a point in the adjustment process at which it may be welfare improving to renege on their commitment to repay their foreign debt. In fact, in the absence of sufficient pre-commitment mechanisms or sufficiently strong sanctions in the case of default, this will happen

\textsuperscript{13} Alogoskoufis (2014) analyzes the effects of budgetary policies on external balance in an endogenous growth overlapping generations model of a small open economy.
with certainty. In such a case, unrestricted financial openness is a non starter, as lenders, anticipating the incentives to default, will never lend. Capital will nor flow from “rich” to “poor” countries. If lenders can credibly commit to limited sanctions in the case of default, international borrowing and lending will not cease completely, but will be subject to foreign debt ceilings for debtor nations.
Appendix

A Numerical Simulation of the Adjustment Paths under Autarky and Openness

The results of a dynamic simulation of both the small open economy model and the two country model are presented in Table A.1 and Figures A.1, A.2 and A.3.\textsuperscript{14}

We consider first the case of a small “poor” economy which, under openness, can borrow at the steady state world real interest rate, as in the analysis of section 4. We assume that the initial output (GDP) per effective unit of labor of this economy is at 70% of its steady state value. The steady state results are summarized in the first two columns of Table A.1, while the adjustment paths are depicted in Figure A.1.

Compared to autarky, financial openness results in an immediate rise in the investment rate and absolute consumption smoothing (see Figure A.1). The adjustment of the capital stock and output (per effective unit of labor) towards the balanced growth path is thus faster than under financial autarky. However, the economy initially runs trade and current account deficits. Gradually, the trade deficits are transformed into surpluses and the stock of foreign debt stabilizes.

As suggested by the theoretical analysis, the steady state capital stock, steady state output per capita, the steady state shadow price of capital and the steady state real interest rate are the same under autarky and openness, although their adjustment paths differ under the two regimes. However, as the theoretical analysis has also concluded, steady state consumption is lower under openness than under autarky, and national income (GNP) is lower than domestic output (GDP). Because of the large initial gap between the autarky real interest rate and the world real interest rate, and the resulting fast accumulation of external debt, the steady state difference between GNP and GDP is about 18% of GDP in this example, while the steady state difference in consumption between autarky and openness is 5.5% of GDP (see Table 1).

The differences in discounted consumer utility between openness and autarky, although positive, are relatively small, confirming the results of Gourinchas and Jeanne (2006), who used a model without adjustment costs for a similar calibration exercise. After 300 periods (years), the discounted utility of the representative household, assuming logarithmic preferences, is only 0.33% higher under financial openness than under autarky. This, as Gourinchas and Jeanne (2006) have argued, is due to the temporary nature of the distortion created by the lower initial capital stock of the growing economy. However, after the first 30 periods (years) of the adjustment, when the “poor” economy is still running trade deficits, the discounted utility of the representative household under financial openness is 14.2% higher than under autarky. This suggests quite sharp inter-temporal tradeoffs. For the first 30 years of the adjustment process the “poor” economy is enjoying welfare benefits in terms of higher consumption than under autarky, and, subsequently it is suffering welfare losses relative to financial autarky.

We next consider the case of a symmetric two country world economy. We assume, as in the analysis of section 5, that the only difference between the two countries is in their initial capital.

\textsuperscript{14}The simulations were carried out in MATLAB, using the DYNARE pre-processor (see Adjemian et al (2011)). The parameter values used were, $\rho=2\%$, $n=1\%$, $g=1.5\%$, $\delta=3.5\%$, $\alpha=2$, $\alpha=0.33$ and $\phi=6$. The qualitative nature of the simulation results is not sensitive to the exact parameter values.
stock and the resulting per capita GDP. The initial output (GDP) per effective unit of labor of the “rich” economy 1 is at 90% of its steady state value, while the initial output (GDP) per effective unit of labor of the “poor” economy 2 is at 70% of its steady state value. The steady state results are summarized in the last four columns of Table 1, while the adjustment paths are depicted in Figures A.2 and A.3.

As suggested by the theoretical analysis, the steady state capital stock, steady state domestic output (GDP) per capita, the steady state real wage, the steady state shadow price of capital and the steady state real interest rate are the same under autarky and openness for both economies, although their adjustment paths differ under the two regimes.

Compared to autarky, for economy 1 (the “rich” economy) financial openness results in a fall in the investment rate and an initial fall in private consumption (see Figure A.2). This is because the real interest rate rises compared to autarky. The adjustment of the capital stock and output (per effective unit of labor) towards the balanced growth path is thus slower than under financial autarky. As a result of the higher savings and the lower investment, the economy initially runs trade and current account surpluses. Gradually, the trade surpluses are transformed into deficits and the stock of foreign assets stabilizes. As the theoretical analysis has concluded, steady state consumption is higher under openness than under autarky, and national income (GNP) is higher than domestic output (GDP).

The opposite happens in economy 2 (the “poor” economy). Both investment and consumption rise, because of the fall of the real interest rate compared to autarky (see Figure A.3). Consumption smoothing is not absolute (as in our “small” economy example) but relative, since the world real interest rate is initially above the steady state real interest rate and declining. In all other respects, the adjustment path of economy 2 resembles the adjustment path of the small open economy that we have already analyzed. In this case, because of the smaller initial gap between the autarky real interest rate and the world real interest rate, and the resulting slower accumulation of external debt, the steady state difference between national income (GNP) and domestic output (GDP) is about 7.3% of GDP, while the steady state difference in consumption between autarky and openness is 2.6% of GDP (see Table 1).

The differences in discounted consumer welfare under openness and autarky are also small, for both countries. For example, for both the “poor” and the rich economy, after 300 periods (years) discounted welfare under openness is only 0.035% higher than under autarky. The reason is than under the assumptions we made in the two country model, the real interest rate falls by less in the “poor” economy after financial openness, than in the one country small economy model, where we assumed that the world interest rate jumps to its steady state value. Again, the benefits are front loaded and the welfare costs are back loaded for the “poor” economy. The opposite applies to the “rich” economy. After the first 30 periods (years), discounted welfare under openness is higher by 4.6% compared to autarky in the “poor” economy, and lower by 3.6% in the “rich” economy.
References


Figure 1
Dynamic Adjustment under Financial Autarky

\[ q = 0 \text{ Steady State Investment Schedule} \]

\[ \dot{q} = f + \psi (g + n + \delta) \]

\[ \dot{k} = 0 \text{ or } c = AK^2 q (g + n + \delta) k \]

\[ c^* = 0 \text{ Steady State Equality between Savings and Investment} \]
Figure 2
The Time Paths of Output and Consumption under Financial Autarky
Figure 3
The Dynamic Adjustment of Investment and Capital under Financial Openness

$q_t = 1 + \phi(g+n+\delta)$

Steady State Investment Schedule

Adjustment Path under Autarky
Adjustment Path under Openness
Figure 4
Consumption and the Trade Balance
in a Small Open Economy
Figure 5
The Dynamic Adjustment of Investment and Capital in a Two Country World

\[ q_t = 1 + \phi(g + \alpha + \delta) \]
Figure 6
Consumption and Trade Balances in a Two Country World
Figure 7
The Evolution of the “Poor” Country Value Functions under Financial Openness and Autarky, and the Default Decision
Figure 8
The Evolution of the “Poor” Country Value Functions under Financial Openness and Autarky, under Optimal Sanctions
Figure A.1
The Adjustment Path of an “Poor” Small Economy under Financial Autarky and Openness

Note: The blue line depicts the adjustment path under financial autarky and the red line the adjustment path under financial openness. In the panel on the Trade Balance and the Current Account, the trade balance is depicted by the blue line and the current account by the red line. See the text and the notes to Table 1 for the simulation details.
Figure A.2
The Adjustment Path of Economy 1 ("rich") in a Two Country Model, under Financial Autarky and Openness

Note: The blue line depicts the adjustment path under financial autarky and the red line the adjustment path under financial openness. In the panel on the Trade Balance and the Current Account, the trade balance is depicted by the blue line and the current account by the red line. See the text and the notes to Table 1 for the simulation details.
Figure A.3
The Adjustment Path of Economy 2 ("poor") in a Two Country Model, under Financial Autarky and Openness

Note: The blue line depicts the adjustment path under financial autarky and the red line the adjustment path under financial openness. In the panel on the Trade Balance and the Current Account, the trade balance is depicted by the blue line and the current account by the red line. See the text and the notes to Table 1 for the simulation details.
Table A.1
Simulation Results for the “Poor” Small Economy and Two Country Models

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Note: The simulations have been carried out in MATLAB, using the DYNARE pre-processor (see Adjemian et al (2011)). The parameter values used were, $\rho=2\%$, $n=1\%$, $g=1.5\%$, $\delta=3.5\%$, $A=2$, $\alpha=0.33$, $\phi=6$. $y^*$ is Gross National Income (per effective unit of labor), $s$ is the savings rate (% of GDP), $tb$ is the trade balance (% of GDP) and $ca$ is the current account (per cent of GDP). Subscript $E$ refers to the steady state (balanced growth path). $y_0$ is the initial GDP (per effective unit of labor). It was set at 70% of steady state GDP for the poor small economy model and the poor economy in the 2 country model, and at 90% of steady state GDP for the rich economy in the 2 country model.